# Truth and conditionals

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# Conditionals

$$\longrightarrow$$

# T-sentences

$$T(\ulcorner A \urcorner) \leftrightarrow A$$

# Reasoning

$$A \therefore B$$

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$$A, A \rightarrow B \vdash B$$

# Rejecting

$$\dashv \lambda$$

# Weakening - I

$$\not\models T(\ulcorner \lambda \urcorner) \leftrightarrow \lambda$$

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$$\not\models T(\ulcorner \lambda \urcorner) \leftrightarrow \lambda$$

$$T(\ulcorner A \urcorner) \models A$$

# Weakening - II

$$A, A \rightarrow B \not\models B$$

# Weakening

$$\not\models T(\ulcorner \lambda \urcorner) \leftrightarrow \lambda$$

$$A, A \rightarrow B \not\models B$$

☹

# Feferman objection

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$$\begin{array}{|l} A \\ \hline \vdots \\ B \end{array}$$

$A \supset B$ ????

# Conditionals

$\supset$

# Conditionals

$$\supset$$

$$\supset \neq \rightarrow$$

# Field

$$A \rightarrow B \models C \rightarrow A \rightarrow .C \rightarrow B$$

# Beall

$$A, A \rightarrow B \models B$$

# Several people

$$T(\ulcorner A \urcorner) \leftrightarrow A$$

# Field again, sort of

$$DA =_{Df} A \& \sim (A \rightarrow \sim A)$$

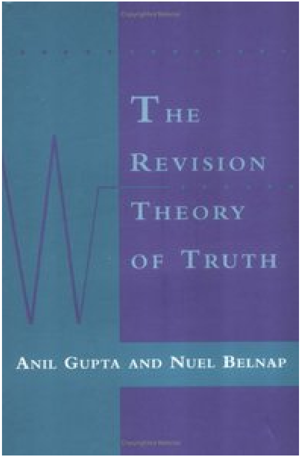
$$\dashv A = \sim D^* A$$

# Roles

Reasoning

Truth-theoretic features

# Revision theory



# Circular definitions

$$Gx =_{Df} A(x, G)$$

# Example

$$Gx =_{Df} (x = a \ \& \ \sim Gx) \vee (x = b \ \& \ Gx)$$

# Hypotheses

$$h \subseteq D$$

# Revising

$$Gx =_{Df} A(x, G) \mapsto \delta$$

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$$Gx =_{Df} A(x, G) \mapsto \delta$$

$$h, \delta(h), \delta(\delta(h)), \delta^3(h), \ldots, \delta^\omega(h), \ldots$$

# Example

$$Gx =_{Df} (x = a \;\&\; \sim Gx) \vee (x = b \;\&\; Gx)$$

| | 0 | 1 | 2 | ... |
|---|---|---|---|---|
| $\emptyset$ | $\emptyset$ | $\{a\}$ | $\emptyset$ | |
| $\{a\}$ | $\{a\}$ | $\emptyset$ | $\{a\}$ | |
| $\{b\}$ | $\{b\}$ | $\{a, b\}$ | $\{b\}$ | |
| $\{a, b\}$ | $\{a, b\}$ | $\{b\}$ | $\{a, b\}$ | |

# Revision theory of truth

$$
\begin{aligned}
T(\ulcorner A_1 \urcorner) &=_{Df} A_1 \\
T(\ulcorner A_2 \urcorner) &=_{Df} A_2 \\
&\vdots \\
T(\ulcorner A_n \urcorner) &=_{Df} A_n \\
&\vdots
\end{aligned}
$$

# Classical logic and T-sentences

$$a = \ulcorner \sim Ta \urcorner$$

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$$a = \ulcorner \sim Ta \urcorner$$

$$\vdash_K \sim (A \equiv \sim A)$$

# Classical logic and T-sentences

$$a = \ulcorner {\sim} Ta \urcorner$$

$$\vdash_K \sim (A \equiv {\sim} A)$$

$$\nvdash_{RT} Ta \equiv {\sim} Ta$$

# It gets worse

$$\vdash_{RT} \sim (Ta \equiv \sim Ta)$$

# Definitional equivalence

$$=_{Df} \quad \neq \quad \equiv$$

# Conditionals for revision theory

$$A \rightarrow B, A \leftarrow B$$

# Conditionals for revision theory

$$A \rightarrow B, A \leftarrow B$$

$$A \leftrightarrow B \; := \; (A \rightarrow B)\&(A \leftarrow B)$$

# New hypotheses

$$h \subseteq F \times V$$

# Semantics

$$M, v, h \models A \rightarrow B \quad \Leftrightarrow \quad M, v, h \not\models A \text{ or } \langle B, v\rangle \in_M h$$

$$M, v, h \models B \leftarrow A \quad \Leftrightarrow \quad \langle A, v\rangle \notin_M h \text{ or } M, v, h \models B$$

# Rules

$$\begin{array}{l|l} & \begin{array}{|l} A^{k+1} \\ \hline \vdots \\ B^{k} \end{array} \\ & A \rightarrow B^{k+1} \qquad \rightarrow\text{I} \end{array}$$

$$\begin{array}{|ll} A^{k+1} & \\ A \rightarrow B^{k+1} & \\ B^{k} & \rightarrow\text{E} \end{array}$$

$$\begin{array}{l|l} & \begin{array}{|l} A^{k} \\ \hline \vdots \\ B^{k+1} \end{array} \\ & B \leftarrow A^{k+1} \qquad \leftarrow\text{I} \end{array}$$

$$\begin{array}{|ll} A^{k} & \\ B \leftarrow A^{k+1} & \\ B^{k+1} & \leftarrow\text{E} \end{array}$$

# Features

$$\models_{RT^+} T(\ulcorner A \urcorner) \leftrightarrow A$$

# Features

$$A =_{Df} B \quad = \quad A \leftrightarrow B$$

# Features

$$Gx =_{Df} A(x, C(Gx \leftrightarrow B))$$

# Logic - Sameness

- $(A \rightarrow C) \supset (A\&B \rightarrow C)$
- $(A \rightarrow B)\&(A \rightarrow C) \supset .A \rightarrow (B\&C)$
- $A \vee B \rightarrow C \supset .A \rightarrow C$
- $(A \rightarrow C)\&(B \rightarrow C) \supset .A \vee B \rightarrow C$
- $(\sim A \rightarrow B)\&(\sim A \rightarrow \sim B) \supset .A$

# Logic - Difference

- $\models ((C \leftarrow B) \leftarrow A) \equiv (C \leftarrow A\&B)$
- $\not\models (A \rightarrow (B \rightarrow C)) \supset A\&B \rightarrow C$
- $\not\models (A\&B \rightarrow C) \supset (A \rightarrow (B \rightarrow C))$
- $A \rightarrow (A \rightarrow B) \not\models A \rightarrow B$
- $(B \leftarrow A) \leftarrow A \models B \leftarrow A$

# Logic - Interaction

$$(A \rightarrow B) \equiv (\sim A \leftarrow \sim B)$$

# Flaws

# Flaws . . .

# Flaws . . . ??

# Flaws . . . ??

$$\not\models A \rightarrow A$$

# Flaws . . . ??

$$\not\models A \rightarrow A$$

$$A \rightarrow B, B \rightarrow C \not\models A \rightarrow C$$

# Flaws . . . ??

$$\not\models A \rightarrow A$$

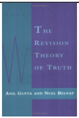
$$A \rightarrow B, B \rightarrow C \not\models A \rightarrow C$$

$$A \leftrightarrow B \not\models B \leftrightarrow A$$

# Seriously?

$$\rightarrow, \leftarrow \;\neq\; \Rightarrow$$

# Roles revisited

| | Reasoning | Truth |
|---|---|---|
|  | $\rightarrow_F$ | $\rightarrow_F$ |
|  | $\rightarrow_{BX}$ | $\rightarrow_{BX}$ |
|   | $\supset$ | $\rightarrow, \leftarrow$ |

# Too complicated

$$M, v, h \models A \rightarrow B \quad \Leftrightarrow \quad M, v, h \models A \text{ or } \langle B, v \rangle \in h$$

# Completeness

$$\models^{\mathscr{D}}_{RT^+} A \;\Leftrightarrow\; \vdash^{\mathscr{D}}_{RT^+} A$$

# Naturally fits into the revision theory

$$Gx =_{Df} A(x, G)$$

# Naturally fits into the revision theory

$$=_{Df} \quad \neq \quad \equiv$$

# Conclusions

- Distinguish roles conditionals play in our theories
- These roles can be used to motivate the addition of conditionals to logics
- Adding conditionals to revision theory fixes one of its problems
- These conditionals fill out the formal and philosophical picture of the revision theory
- Our earlier distinction can be used to defend these conditionals against objections

# Thank you

. . . to you, the audience.
. . . to Shunsuke Yatabe for inviting me.
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# Field quote

Field says the following of the strong Kleene material conditional.

> *But while [the material conditional] does a passable job as a conditional in the presence of excluded middle, it is totally inadequate as a conditional without excluded middle: with $\supset$ as one's candidate for $\rightarrow$, one wouldn't even get such elementary laws of the conditional as $A \rightarrow A$, $A \rightarrow (A \vee B)$, or the inference from $A \rightarrow B$ to $(C \rightarrow A) \rightarrow (C \rightarrow B)$. . . . The lack of a conditional (and also of a biconditional) cripples ordinary reasoning.Field (2008, 73)*

Field says that his conditional "enables us to come much closer to carrying out ordinary reasoning" than the strong Kleene material conditional does.Field (2008, 276)

# Feferman quote

"[N]othing like sustained ordinary reasoning can be carried on in [strong Kleene logic]." Feferman (1984, 95)
The whole quotation is emphasized in the original.

# Beall quote

"The question is whether we have detachable Tr-biconditionals (i.e., ttruth-biconditionals). If we do, then such biconditionals are not our usual material biconditionals, as noted above. I think that we do enjoy detachable Tr-biconditionals. . . ." (Beall, 2009, 26)

# Bibliography

Beall, J. (2009). *Spandrels of Truth*. Oxford University Press.

Feferman, S. (1984). Toward useful type-free theories. I. *Journal of Symbolic Logic*, 49(1):75–111.

Field, H. (2008). *Saving Truth from Paradox*. Oxford.